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CONCERNING THE ANCIENT GREEK IDEAL OF THEORETICAL THOUGHT

1. The logical reconstruction of the theoretical systems of ancient science permits an appraisal of the character of the theoretical achievements of antiquity in a new way. The reconstruction of the structure of ancient logical systems, begun by J. Łukasiewicz and his pupils, can be compared with the reconstruction of the Euclidean geometry during the intensive development of geometry in the 19th and 20th centuries, first of all in the works of D. Hilbert. Extremely interesting results, in particular, were obtained concerning the method of analysis, as it was realized in ancient texts [1], [2].

It seems that historical retrospection can be the aim of logical advancement only to a small extent, as the reconstruction of Euclidean geometry has been only a subordinate result of the development of contemporary geometry. To make the picture of the past more complete, it would be useful to supplement the reconstruction of the structure of ancient theoretical thought with an analysis of general notions about the ideal of scientific thought prevailing in the epoch of creation of the axiomatic method. This is the main task pursued in the present paper.

The premises I am starting from can be characterised in this way: I doubt whether the intimate mechanism of reasoning differs essentially in different societies and in different epochs. That's not we are talking about, to find differences between "ancient Greek syllogisms" and "ancient Indian syllogisms". The logical mechanism of solution in theoretical, in particular mathematical, tasks is generally the same. At least more obvious are the differences of a higher order manifesting themselves when a solution of problems satisfying a certain society in a certain epoch ceases to satisfy another society in another epoch. For instance, the result known as the "Pythagorean theorem" was obtained by the ancient Greeks as well as by the ancient Chinese. From the point of view here formulated it is of no importance how this result was being obtained by the Greeks or by the Chinese. It is important to know why solutions which had satisfied the Chinese didn't satisfy the Greeks.

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Needless to say, such an analysis is in any case important not so much for the history of science as for the self-evaluation of modern scientific knowledge. Our criteria of selection of acceptable theoretical constructions, including our criteria of exactness and demonstrativeness, are historically dependent and historically confined, and self-knowledge is possible only in comparison with other scientific communities.

2. Achievements of contemporary comparativistics enable one to maintain that extremely archaic structures of outlook and notions, frequently descending from common Indo-European roots, were, firstly, unexpectedly complicated and advanced, and, secondly, had a lively cultural background in ancient society to a more considerable extent than they have been thought to have. Well, the idea of the world fire as the universal substance and the eschatology of world fires, inherent in the ancient Indo-European thought, still lived until recent times in the thought of many people. The stability and habitualness of this idea explains well-known passages in Heraclitus. (Here other ancient ideas, particularly Stoic ones, could also be mentioned.) Democritos' idea that the image of thing is created between a thing and the eye as the result of emanations from the eye to the thing, will stay unintelligible until we take into account the fact that for the archaic outlook "to touch by eye" was not sharply differentiated from touching by hand (many ethnographic materials witness to this). That's one reason why the ideal of theoretical knowledge formed in ancient Greece ought to be considered against the background of archaic notions.

It is universally recognized that the social system of democracy in the Greek politics was a factor of great importance, which determined a new understanding of cognition. (See, in particular, [3], p. 18 and ff.) From following custom as established by God (*θέμις* – "supposed") the society goes over to following law as being humanly established (*νόμος* – "law"). This change in outlook manifested itself, particularly, in the desacralisation of the cosmological symbolism in Greek architecture (see [4], [5], [6]).

It is necessary to mention that, as was ascertained by S. Vikander, in the ancient Indo-European community there already existed a strict opposition of the practical sphere to the sacral sphere (for example, continuous sacral exchange between members of the community had been opposed to their practical dealings; mythological personages embodied both the sacral idea and its physical action – cf. Indra and Vayu in the Indian mythology). From another side, desacralised sphere in the ancient Greek conscious incompletely corresponds to the modern notion of the "secular" world.

In archaic consciousness the world was thought to be an action filled by some inner sense, an action which as modelled in ritual performances and prophecies. World was thought of as the action, *θέā*, performance, spectacle, in which man takes part and at the same time apprehends, sees, observes it. (Common Indo-European *$\sqrt{dhāu}$; compare Sanscrit *dhīḥ* – "thought", "wisdom", *dīdhyē* – "I observe", "perceive", "think"). The most important act in this model of the world-spectacle was to put objects in their own places, i.e., to put the world in order (*θέσις* – action of putting something in its own place, without which the world, *θέā*, is unthinkable). Indicating or drawing lines is the most important function in the regulation of the world. The Indo-European word *$\sqrt{di\hat{k}}$ means "imperative legal order", "instruction"; from here we have in Sanscrit *diṣtih* – "designation", *diç* – *dik* – "direction", "designation", and in ancient Greek *δικέ* – "world legal order". On this ground the notion of demonstration – *δείκνυμι* (I), – "to make somebody to see" (see [7], [8], [3]) was first developed.

Desacralisation of cultural life showed itself, particularly, in the liberation of ideas about "indication" as a means of understanding the sense of cosmic *θέā* from ritual load. A. Szabó showed that the terminology of the geometric doctrine of proportion has its origin in the theory of music [9]. Probably in this case more general reasons are to be considered, than the influence of Pythagoreans. Desacralised, secular art, both architecture, theatre, and music, together with philosophy and mathematics, was confronted with the ritual, sacral sphere of culture. The general notion for the whole secular sphere was *θεωρία* as "action of seeing", "looking", and *θεώρημα* as "an object of sight", "something that one sees", "spectacle" (the same terminology is used in sacral sphere: compare *θεωρός* – "ambassador on prophecies and games", *θεωρίς* – "Sacral boat, brought *theoros* to Delos").

In connection with this the verb *δείκνυμι* – "show", "indicate" – too attains the significance relatively free from the sacral sense, but at the same time common for the whole secular culture. A. Szabó, who discovered the evolution of this notion, notes three meanings of the term $\delta\epsilon\acute{\iota}\kappa\nu\upsilon\mu\iota$: [1] "to show, to make somebody to see, to indicate; [2] to explain; [3] to demonstrate. In the beginning period, I believe, there were not three but one and the same meaning. It is interesting that Socrates, according to Xenophon, raised the traditions of his $\delta\epsilon\acute{\iota}\kappa\nu\upsilon\mu\iota$ to Odysseus because of his ability to convince ("to show", "to demonstrate by words"), based on what is generally recognized and unquestionable. In this case one can see the most ancient scheme of the future axiomatic constructions. I believe that the criteria of the correctness of the $\delta\epsilon\acute{\iota}\kappa\nu\upsilon\mu\iota$ for the first time didn't permit proper geometrical demonstrations from verbal explanations of the essence of the world.

To the end of antique epoch Lucian ridiculed the groundless explainings of natural philosophy:

And doesn't it demonstrate the stupidity and full ignorance of philosophers speaking about things not so clear but insisting on their correctness and denying the contrary view to have any significance, they hardly do not swear, that Sun is scorching globe, that Moon is populated, that stars drink water scooped by Sun from the sea as in the well rope, and distributes it between them equally? ((10), p. 277–278).

Thales from Miletus belonged to such natural philosophers, to whom probably belonged the first geometrical demonstrations ((10), p. 67). I believe that their criteria of the correctness of the $\delta\epsilon\acute{\iota}\kappa\nu\upsilon\mu\iota$ for the first time didn't permit to separate proper geometrical demonstrations from verbal explanations of the essence of the world.

But sacral "explaining" and desacralised "indication by words" are strictly opposed. Very interesting from the point of view of the development of semiotic ideas is the substantiation of legitimacy of the two types of understanding of the world given by Plutarch, a near contemporary to Lucian.

"But, as I suppose, nothing prevents both scientists and speculators from being right, because the one explains the causal connection and the other the purposive one.... Those who think that finding the causal connection in some events they prove these events couldn't be signs, overlook that in reasoning in such a way they deny existence not only of divine signs but of any artificial signs as, for instance, signs made by iron disks or by fire, or the definition of time by the length of the shadow of sun dials." ([11], p. 200.)

3. The second progress of great importance in ancient consciousness was the separation of the indication of empirical data from the

indication of abstract essences and mentally performed actions. It is this progress that the formation of proper geometrical demonstration is connected with. The activity of Eleatic philosophers had outstanding significance for Greek mathematics, very well shown by A. Szabó [9]. *θεώρημα* becomes an abstract construction, *θεωρια* a mental construction generating abstract objects.

This process, connected with geometrisation of the Greek mathematics, is well investigated. Strong and weak sides of the mathematical methods of ancient Greeks are connected with their ideals of acceptability of mathematical constructions that found their expression in the geometrisation of mathematics. Geometrical methods were confined by the fact of incommensurability, the discovery of which, according to Szabó, led to a "new technology of demonstration and to antiempirical transformations alien to visual methods" ([9], p. 287). Along with the feat of paradoxes of infinity and continuity generated by these methods, they paralyzed the possibilities of Greek theoretical thought, as was excellently shown by V.F. Kagan when he analyzed the legacy of Archimedes [11].

Judging from these generally recognized statements, the assertion can be formulated that the ancient Greek ideal of theoretical knowledge is derived from the notion of thought as a process of construction, a notion that ought to be distinguished clearly from the habitual notion in European science of thought as a calculation. In addition to this thesis I want to put forward some considerations about another step which is in my opinion of first rate significance for the formation of the ancient ideal of demonstrativeness. This matter is the opposition of analysis and synthesis, which for the Greeks was much more significant than the opposition of deduction and induction.

4. It is clear why induction is not opposed to deduction by Aristotle but considered (rather in passing) in the second part of his *Analytics* as a special case of syllogism: as ancient scientific thought didn't want to deal with infinities, induction can be identified with syllogism. But why did Aristotle avoid the term "logic", preferring instead of it "analytics"? Are there any other grounds except the ambivalence of the term λόγος which meant both "thought" and "word"? After all λόγος has had a long tradition up to this time! If the ancient image of scientific knowledge is oriented to mental construction, than, it seems, it is more natural to connect it with the notion of synthesis than with analysis.

The terms "analysis" and "synthesis" have in ancient Greek language the same sense as in the modern intuition: analysis is the separation of the whole into parts; synthesis is the joining of parts in a whole. But it is the word "synthesis" that is connected etymologically with the idea of theory: σῦνθεσις is derived from the notion of θέσις. The joint bringing up of some of "theses" to be considered as a whole (ςὺν – "with") was an action which from the point of view of the ancient Greek outlook had to be of much greater importance than a simple "separation of connections", "untieing", "solving" (latin "resolutio": "ἄν – ἄλυσις" – from "ἅλυσις", "chain", "connection").

A strong impact on the ancient outlook was made by Greek writing, which was the first one to realize the phonetical principle. In particular, this paradigm of significant whole, formed of elements, each of which has no significance alone, lies at the bottom of Greek atomism. This paradigm relies upon the image of the connection between words and letters. Of course, in order to make words of letters, it is necessary to divide the word in sounds; but the idea that the world consists of some first elements, ἀρχαι, is very ancient alone, and mystery, as it seemed, consists in how the whole is made of ἀρχαί.

Original ideal δείκνυμι just coincided with mental construction as connecting the parts in a whole by means of verbal explanation, i.e. with σύνθεσις. True change comes in the time of Plato and is connected with his opposition to arbitrary explanation – substantiation, characteristic of natural philosophers; the substantiation that excludes certain possibilities and hence explains why the rest is necessary. Analysis precludes splitting a situation in something similar to "possible worlds" with a view of discovering among them the impossible ones and leaving only one possible, i.e. necessary, one. To judge from some ancient testimonies, the analytical method suggested by Plato consisted in accepting the unknown as a known. (See [12].) This means, in a certain sense, the possible, i.e. the unknown, "equality of rights" granting with the actual. From here takes its beginning Aristotle's understanding of necessity as impossibility to be otherwise. First a certain "to be otherwise" is postulated, and then its impossibility is stated. As was shown by J. Hintikka and U. Remes, the Greek analysis, as it was understood as a "resolution", had indeed been a combination of analysis and synthesis. It is interesting, too, that in accepting the unknown as known, according to O. Becker [15], Greeks were acquainted in a yet early period of development of mathematics with a form of semi-arithmetic, semi-geometrical method of solving arithmetic problems with the help of stones, perhaps of different colours and sizes. The high method of analysis has, consequently, its sources in the contemptible technology of counting. Nevertheless, essentially it means becoming aware of the splitting of the world in possible, impossible, and necessary situations, historically connected with the equality of rights of the known and the unknown. Thereby the rule of contraries is realized as a tool of theoretical analysis. This separates the ideal of apodictic necessity of theoretical demonstration from abstract natural-philosophical speculations. In this case the notion of thought as construction goes to background, but remains a ground of the ideal of theoretical knowledge.

But where is the source of Aristotle's dislike to terminology derivative from the word "λόγος"? In his *Analytics* Aristotle explains how logical and dialectical conclusions differ from proper analytical ones in containing unreliable, probable, "verbal" knowledge because they are based only on unanalyzed but commonly recognized theses. So, it seems that analytics is opposed to logic: dialectics is a reflection of an opposition of science to sophistics as verbal art; and as the point of departure in the term "λόγος" the meaning "word" is taken instead of the meaning "thought".

From Socrates undoubtedly begins that turn in the ancient Greek outlook which led to the logico-philosophical realisation of the ideal of analytically necessary knowledge. Alongside with this, Socrates's ideal of demonstrable knowledge looks still quite archaic. Maieutics of Socrates generated in term "induction". (But this is by no means the modern sense of the term: ἐπάγωγί of Socrates is, in essence, a prototype of deduction, because in search of counterexamples Socrates addresses data obviously known to the speakers beforehand.) To divert from it counterexamples taken from experience, we have a number of abstract possibilities – a scheme of the analysis. But it is characteristic that Socrates, according to Xenophon, treated abstract demonstrations in geometry sharply negatively as having only pragmatic, applied functions, the functions of counting. For this thinker first brought the problem of man to the centre of philosophy where its place is logically; and along with it we clearly see here the combination of courageous ideas of Socrates with his general conservatism, in this case in mathematics. This may appear symptomatical, because the Stoic tradition grew from Socratic schools as well as from the traditions of Platonists and peripatetics, but Plato and Aristotle didn't accept the anti-deductivism of Socrates in the sphere of mathematics. Along with this, it is the Stoic tradition that we are obliged to for the term "logic".

In Stoic philosophy we find an advanced semantics and, in particular, an opposition between a sign or name ("significative") and sense ("denoted"). But "λόγος", for the Stoics, is to the same extent "notions" as "words". Basing our views on later European traditions, we frequently modernize the meaning of the term "λόγος" and the ancient ideas of the name and the sense. Meanwhile, the archaic thought opposes "the name" and "the sense" in a way different from the modern one. In ancient Chinese philosophy the opposition of "name" and "business" ("min" and "sin") is associated with the pair "question" – "answer": name is the symbolic expression of certain circumstances, demanding an interpretation (a "business" or "sin") in the same way as a question. Indicating the sense of a given situation is the interpretation, in the same way as an answer (a "name" or "min"). That's why it is deeply false to identify the relation of "min" and "sin" with the relation of subject and predicate of the European tradition, although a distant connection is possible here. The essence of a subject is clarified by the practice of canonic questions and answers in puzzles of mythological content of different people. (For instance, Siberian Evenc-hunters ask: "What's the hole made in the wild deer's skin?". It is necessary to answer: "the sky". First there is a "business", second there is a "name".

Echoes of similar very archaic opposition we found in Heraclitus: "So, the name of the bow – the life, and the business – the death" ([14], fragment 48). The term "λόγος" does not come from an opposition of the inner sense and language expression; in Stoic philosophy, though acquainted with such opposition, λόγος still remains the inseparable "word-thought". Peculiarities of the sense of the term λόγος in the ancient texts were analyzed for the first time by A. Szabó, who came to the conclusion that originally it was a designation of each of two numbers, which were the limits of a διασθημα-interval in musical theory. Hence λόγος is a proportion, or a relation of two extreme numbers (proportion – ἀναλογία – "equality according to logos"). To the original senses of λόγος – "speech", "language", "thought" Szabó adds "number", "a number or community of things". The etymology of the word is related to that of λέγω, which means "to gather", "to choose", "to speak", "to enumerate" deriving from the Indo-European word $*\sqrt{\bar{l}\bar{e}g}$ – "to gather"; from here λόγος – "speech", "story", "intelligence", "calculation" [8]. (Compare Latin – legion = "people, gathered by words of command"). We might say that λόγος means "word-notion" in the sense of "set" (collection). It is the tradition that dominates in Pythagorean and Heraclitean philosophy, that Stoics return to. The term λόγος and the idea of the word-set, developing in the technology of counting and theory of proportions, generates sooner the practical and the theoretical *logistics* ([17], p. 132).

Unlike the traditions of Plato and Aristotle, which were classical for the ancient mind and in a definite way influenced the formation of logico-philosophical paradigms of axiomatic method in Greek science, the Stoic tradition is more connected with practice of rhetoric and logistical practice of calculation. (To nobody among Greek authors, except the Stoics, did it occur to calculate duration of the "great year", which exceeded in duration the number 10,000, which was for the Greek calculators the limit of great numbers!) One can argue with the statement of Łukasiewicz that Aristotle for the first time started using variables in the history of science (in logic); to an extreme extent, in the mind of ancient scientists verbal symbols used in syllogistics didn't differ from those accepted in geometry. As for the Stoics, they created the propositional calculus, where the same words "first", "second", etc. are used that played the role of variable quantities in the mathematics of Diophantos, too. (History of science knows arguments in favour of the first sign of variable ∫ α having its origin not from ἀριθμός "number", but from πρῶτος "the first"; see ([17], p. 146).

So, one can think that the opposition of analytics and logic goes together with the opposition of analysis as a choice between possibilities and verbal art, also with the opposition "theoretical analysis and the art of counting". Traditions of Babylonian algebra didn't fade in ancient Greek mathematics, and in its late period they were advanced by Diophantos. However, they didn't determine the general complexion of scientific thought with axiomatic method characteristic to it. Analogous tendencies in the field of logico-philosophical realizations of the ideal of theoretical thought are represented by the Stoics, whose legacy to the same time it is not characteristic of its main lines of development.

Because ancient influences seem to have reached Rome through the

Stoics more than through other schools and because of the incompetence of Roman translators, in Latin and, particularly in the Western European tradition of the Middle Ages the understanding of differences between logic, dialectics, and analytics inherent in ancient Greek tradition seems to have been lost. In later European university tradition the position is intensified by the fact that in courses of dialectics formal logic was usually taught, while in courses of logic the subject matter was something like epistemological commentaries on it. All this lead to wider understanding of the term 'logic' (for example, in Hegel). But the gist of the matter is the fact that the reception of logico-analytics in European Middle Ages was connected with entirely different general conceptions of knowledge and paradigms of demonstrativeness.

5. To compare ancient ideals of theoretical knowledge with modern ideas of demonstration, we can formulate the following statements.

(a) Even though the analytical method, as the ancients understood it, accepted the supposition of the completed character of certain initial constructions, and stood in opposition to synthesis as 'pure' construction, the main paradigm of thought as construction was preserved. Truly speaking, the idea of analysis alone – which meant to accept the unknown, i.e. the possible, as existing – is in conflict with idea of construction. Aristotle becomes clearly aware of this difficulty: on the one side, analysis must start from existing objects; on the other side, the initial definitions are what opens to us qualities of objects. Hence Aristotle's demand of definitions as constructions which at the same time characterize an object and prove its existence ("definition" as "statement, explaining why the thing is" [18], 93b).

(b) Modern methods for examining the demonstrativeness of scientific constructions are based on the idea of *logical calculations*. Under the incomparably higher capacity of these methods the loss of the "naive" idea of demonstration as construction is rather sad in some respects. It can be mentioned that modern logic with its mathematical ideas differs at very essential points from the original confidence of Hobbes, that thought is in essence the same as addition and substraction: Boole's algebra differs from the "usual" addition and multiplication in that it does not satisfy group axioms. Meanwhile, the notion of construction in a sense similar to the ancient one satisfies the group axioms in a way similar to the notion of the most fundamental operations of theoretical thought. Thus the modern ideals of demonstrativeness in knowledge are compatible with the notions of mental constructions.

(c) Modern notions of analytical truth, which are derived from the idea of logical calculation and which go far beyond the idea of "naive", intuitive character of analysis and synthesis, all still rest on the basis of ancient paradigms. It seems desirable to separate the ideas of the analytical and the synthetical from the ideas of deductive and factual (inductive) knowledge.

So, we can start from an intuition of analysis as a mental action, consisting in the construction of the new properties of given objects, and of synthesis as a construction of new objects with given properties. If the aim of the mental constructions is to build sets, then analytical mental action consists in indicating the way the elements of a given set are formed, or distinguishing sub-set from some basic set by means of forming general properties of the elements of the subset. Synthetical, in this sense, will be mental action which consists in indicating the condition or the parameter (i.e. the property) a given set satisfies. From here, under additional limits we may proceed to the commonly accepted definitions in logic and mathematics.

(d) In ancient science with its axiomatic method, and in ancient logic, which can be named an ideology of the axiomatic method, the rupture between demonstration and explanation did not exist, and a good definition was even considered as an explanation of bases and causes. In modern thought a rupture emerged between demonstration and explanation (description of the observed necessary consequences of certain statements and explanation of their sense). It would be interesting to discover to what extent the ideal of demonstration as logical calculation, which has dominated the mathematized part of modern natural science, is responsible for this.

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